EXPERIMENTAL INVESTIGATION OF THE EFFECT OF VELOCITY LAG

OF PARTICLES IN A SUPERSONIC GAS STREAM

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One of the important problems of contemporary gasdynamics of multiphase systems [1] is to investigate the effect of velocity lag of particles, accelerated in supersonic streams, and to determine their aerodynamic drag coefficient C_D, an accurate value of which is required, in particular, to calculate the loss of specific momentum due to the high-speed nonequilibrium expansion process of a two-phase mixture in the nozzles of contemporary solid-rocket engines [2, 3]. However the significant divergence in the available experimental data on determining C_D and on the dependence $C_D = f(\mathrm{Re}, M)$, proposed by different authors (see, e.g., [1]), makes it difficult to calculate the drag coefficient, and also to use for this purpose the "standard" curve obtained for C_D for a sphere in an incompressible liquid, and makes it desirable to conduct further experiments based on new, more sophisticated methods of diagnostics.

This paper describes an investigation of some of the features of the phenomenon of velocity nonequilibrium of a gas and particles, arising when a two-phase flow is accelerated in a supersonic nozzle, and also when there is transition through a normal density shock, and when the drag coefficient C_D of fine particles is measured over a wide range of Reynolds and Mach numbers, using the laser-Doppler method of velocity measurement.

1. It is known that the drag coefficient C_D of a spherical particle moving in a flow of viscous liquid or gas can be determined from the equation of motion, which can be written in general in the form [1]

$$\frac{4}{3}\pi r_r^3\rho_r\frac{dv_r}{dt}=\frac{1}{2}C_D\rho\pi r_r^2|\mathbf{v}-\mathbf{v}_r|^2-\frac{4}{3}\pi r_r^3\frac{\partial p}{\partial z}+\frac{1}{2}\frac{4\pi}{3}r_r^3\rho\frac{d}{dt}(\mathbf{v}-\mathbf{v}_r)+6r_r^2\sqrt{\pi\rho\mu}\int\frac{\frac{d}{d\tau}(\mathbf{v}-\mathbf{v}_r)}{\sqrt{t-\tau}}d\tau+\sum F_m, \tag{1.1}$$

where p, ρ, v and μ are the pressure, density, velocity, and viscosity of the gas; r_r, v_r, and ρ_r are the radius, velocity, and material density of the particle; ΣF_m are the forces applied from the external potential field. A simple estimate shows that in the experiments described (when solid particles with $d_r \geqslant 10\mu$ are moving in the nozzle) the maximum value of the force due to the pressure gradient in the nozzle is $F_p = {}^4/_3\pi r_r^3(\partial p/\partial z)$, and the gravity force of particles is significantly less than the aerodynamic drag force $F_D = {}^1/_2 C_D\rho\pi r_r^2(\mathbf{v}-\mathbf{v}_r)^2$ (in this estimate C_D is determined from the "standard" drag curve of [1], and $(v - v_r)$ is determined from Fig. 1). The terms containing the combined mass and the Basset force can also be neglected, since the latter are appreciable only when the particle density is on the same order as the density of the particle medium being accelerated [1]. Then the equation of motion (1.1) is simplified appreciably:

$$\frac{4}{3}\pi r_r^3\rho_r\frac{dv_r}{dt}=\frac{1}{2}C_D\rho\pi r_r^2|\mathbf{v}-\mathbf{v}_r|^2. \tag{1.2}$$

Hence, it can be seen that the characteristic relaxation time when the aerodynamic drag force acts on a particle is $\tau_D = 4\rho_r d_r/3C_D\rho(\mathbf{v}-\mathbf{v}_r)$, and it increases linearly with increase of density and particle size. From Eqs. (1.2), taking into account that the flow is steady $(dv_r/dt = v_r\partial v_r/\partial z)$, one can obtain an initial relation for the drag coefficient

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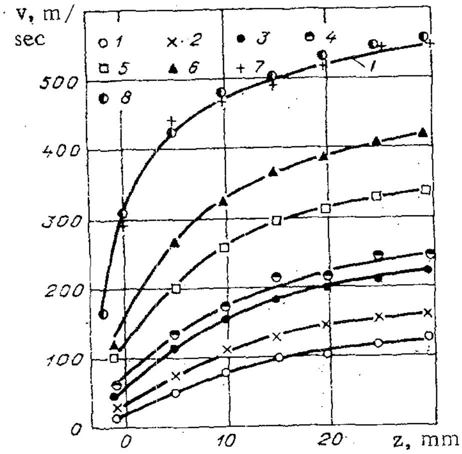

Fig. 1

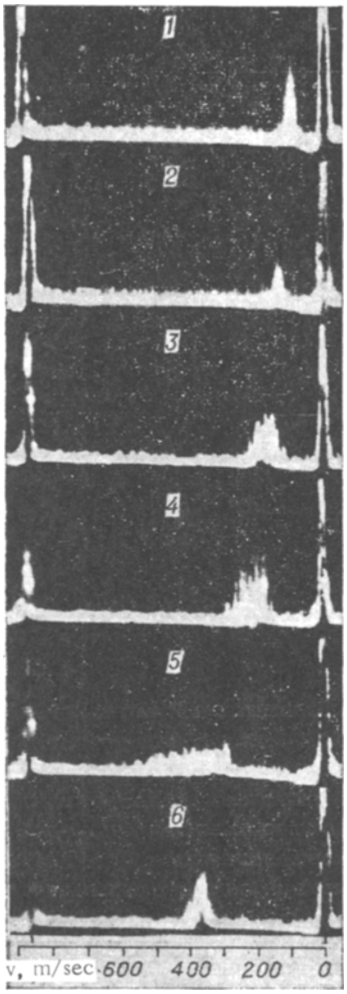

Fig. 2

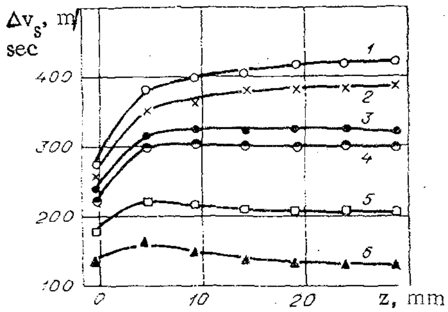

Fig. 3

$$C_D=\frac{4\rho_r d_r v_r}{3\rho\,(v-v_r)^2}\,\frac{\partial v_r}{\partial z}. \tag{1.3}$$

It follows from Eq. (1.3) that to determine C_D, and also the Reynolds number $Re = \rho(v - v_r)d_r/\mu$ and the Mach number $M_s = (v - v_r)/a$ (a is the speed of sound in the gas), which determine the flow conditions, in addition to the values ρ_r, d_r, v_r we must know the gas parameters (ρ, v, μ, T). Here it should be noted that the experimental data refer to the case where the dust particle concentration is quite small, i.e., to conditions where the distance between particles in the flow is a minimum of two orders greater than their dimension, i.e., $l \geqslant 10^2 d_r$ (the "individual" particle regime); then the ratio of the particle mass flow rate to the gas flow rate $\varepsilon = Q_r/Q$ will always be $\leqslant 1\%$. Therefore, the effect of the particles on the gas parameters and their interaction between themselves under these conditions can be neglected, and one can use the gas parameters in the nozzle necessary to compute C_D, M_s and Re, obtained by computation for the "pure" flow.

2. The tests were carried out on a facility with steady supersonic two-phase flow, generated by means of a two-dimensional conical nozzle. The dimensions of the throat section were: height, 2.6 mm; width, 20 mm; angle of opening, 11°. The particles investigated were introduced into the gas stream at a distance L = 300 mm upstream of the throat by means of a special powder device, which produced a uniform supply of powder–air mixture for a period of 10-40 sec. In the experiments a wide set of different particles was used, whose properties are given in Table 1.

The main mass (up to 90%) of all the bronze and lucite particles used had a near-spherical shape. For all the fractions the distribution of particle number with size $dN(r_r) =$

TABLE 1

No.	Material	Density ρ_r, g/cm3	Dimension range $K = d_{max}/d_{min}$	Average size d_{av}, μ
1	Bronze	8,6	3	25
2	"	8,6	1,5	80
3	"	8,6	1,7	190
4	Lucite	1,2	4	200
5	Aluminum	2,7	8	15
6	Lycopodium	0,5	1	25

$f(r_r)dr_r$ was known, determined using a microscope. The value d_{av} in Table 1 denotes the diameter, corresponding to $f_{max}(d_r)$; d_{max} and d_{min} are the maximum and minimum values of the particle diameters, determined at half-amplitude for the given distribution function $f(d_r)$. An estimate of the average particle concentration in the gas stream was based on the known mass flow rates of the gas and the particles, and another estimate was made using a laser-visualization method in conditions where radiation scattered from individual particles could be recorded [4, 5].

The values of density ρ, temperature T, and viscosity μ of the gas along the nozzle were calculated from a one-dimensional model of the steady-state isentropic flow from the given stagnation parameters (determined in the forechamber, p_0 = 8 atm, T_0 = 260°K) and the nozzle profile. The velocity of the gas and the particles was determined by a laser-Doppler method [4, 6]. To measure the gas velocity in two-phase flow we used a signal scattered with smoke particles, introduced simultaneously along with the test particles into the flow by means of a special generator [7]. The validity of using this method was demonstrated by means of control tests, as follows. First, a Pitot tube was used to measure the gas velocity in the dust-free flow v_1 (see Fig. 1, 8, the data v_1), and then only smoke particles were introduced into the flow and a laser-Doppler method was used to measure their velocity v_2 (see Fig. 1, 7, the data v_2). A comparison of the results of these measurements with each other and also with the data for the velocity of the "clean" gas calculated from one-dimensional relationships (see Fig. 1, curve 1) will tell us that the velocity of the smoke particles coincides with the velocity of the gas in the stream, to within the error of measurement of v by the laser-Doppler method, $\leqslant$ 1-2%. (Here one should note that the mass flow rate of the smoke particles Q_r will always be $\varepsilon = Q_r/Q \leqslant 1\%$.)

The subsequent experiments showed that when the test particles are introduced into the flow in conditions where their mass flow rate also did not exceed 1%, the velocity of the smoke aerosols (and therefore, also the gas velocity) did not diminish in comparison with the pure flow, which is additional evidence that it is possible to neglect the influence of particles on the gas when the mass fraction of condensate $\varepsilon \leqslant 1\%$, and it confirms that it is valid to use the values of ρ, T, and μ for the gas, obtained by calculation, in computing C_D, Re, and M_s.

3. The measurements of particle velocity by the laser-Doppler method were carried out on the nozzle axis at points distant z = 0, 5, 10, 15, 20, 25, 30 mm from the throat section. The results of these measurements are shown in Fig. 1, where curve 1 refers to bronze particles with $d_{av} = 190\ \mu$, $\rho_r = 8.6$ g/cm³, 2 refers to bronze particles with $d_{av} = 80\ \mu$, $\rho_r = 8.6$ g/cm³, 3 refers to bronze particles with $d_{av} = 25\mu$, $\rho_r = 8.6$ g/cm³, 4 refers to lucite particles with $d_{av} = 200\ \mu$, $\rho_r = 1.2$ g/cm³, 5 refers to aluminum particles with $d_{av} = 15\ \mu$, $\rho_r = 2.7$ g/cm³, and 6 refers to lycopodium particles with $d_{av} = 25\ \mu$, $\rho_r = 0.5$ g/cm³ (the notation 1-6 of Fig. 1 corresponds to the notation of Figs. 2-4 and Table 1). Figure 2 (z = 15 mm) shows typical oscillograms illustrating the variation of the Doppler frequency shift $\Delta\nu$ and the width of the spectrum of recorded scattered radiation $\delta\nu$ as a function of the parameters of the particles introduced into the stream: the average diameter d_{av} and the material density ρ_r, and also the scattering of particles according to size d_{max}/d_{min}. For two fractions of bronze particles ($\rho_r = 8.6$ g/cm³), which had quite a narrow size spectrum ($d_{av} = 190\ \mu$, $d_{max}/d_{min} = 1.7$ and $d_{av} = 80\ \mu$, $d_{max}/d_{min} = 1.5$), and also for the lycopodium particles (d_r = 24-26 μ, $\rho_r = 0.5$ g/cm³), the half-width of the recorded spectrum $\delta\nu \simeq \delta\nu_a$, where $\delta\nu_a$ is the instrumental half-width of the laser-Doppler system, and depends on the system geometry and the particle speed. The shape of the scattered radiation spectrum was symmetrical, as a rule,

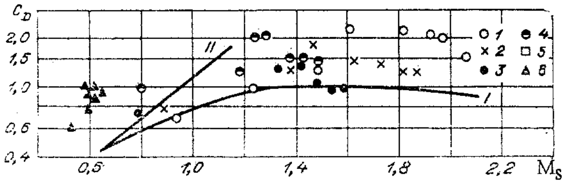

Fig. 4

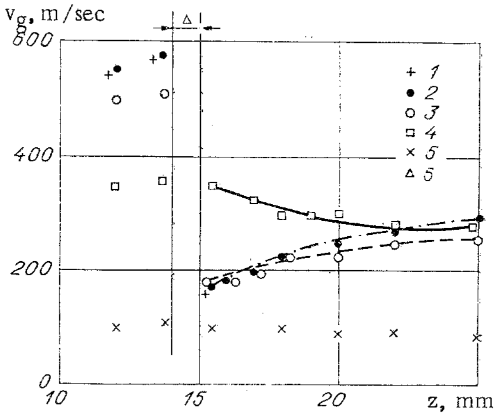

Fig. 5

and from the position of the maximum $f_{max}(\nu_D)$ we could determine the velocity corresponding to d_{av} for the given particle fration. For particles whose size lay in a wider range (particles of lucite $d_{max}/d_{min} \simeq 4$ and aluminum $d_{max}/d_{min} \simeq 8$) the width of the radiation spectrum was considerably increased and was several times larger than $\delta\nu_a$. Thus, for example, for lucite particles the velocity scatter was $\Delta v \simeq 140$ m/sec, and for aluminum particles it was Al$\Delta v \simeq 230$ m/sec (the region from $v_1 \simeq 280$ m/sec to $v_2 \simeq v_{gas} \simeq 510$ m/sec). In this case the velocity of the particles corresponding to the most probable size d_{av} was determined from the maximum in the distribution function $f(v_r)$ for the number of particles with velocity $dN(v_r) = f(v_r)dv_r$. The function $f(v_r)$ was constructed from the corresponding laser-Doppler spectrum under conditions where the radiative pulses from individual particles were recorded [5]. It should be noted that the maximum deviation from the average values of velocity v_r, shown in Fig. 1 and obtained from the results of several tests ($\sim$10), did not exceed 1-2%.

On the basis of the results of measurements of particle and gas velocity (see Fig. 1) at each point of measurement we calculated the quantities $\Delta v_s = v - v_r$ and also $M_s = (v - v_r)/a$ and $Re = \rho d_r(v - v_r)\mu$. The nature of the variation in the velocity acquired by the particles along the nozzle can be followed in Fig. 3, which shows the relation $\Delta v_s = v - v_r = f(z)$ for the various particles. It is clear that the form of the function $\Delta v_s = f(z)$ must depend both on the nature of the variation of the gas velocity along the nozzle $v = f(z)$, and also on the characteristic relaxation time for the particle $\tau_D = 4\rho_r d_r/3C_D\rho(v - v_r)$, which is determined by the properties of the accelerating particles d_r, ρ_r, and the force of interaction between the gas and the particle. It can be seen from Fig. 3 that the largest particle velocity lag Δv_s lies in the region z = 5 mm, located immediately downstream of the nozzle throat, where the gas acceleration $(\partial v/\partial z)$ is a maximum, and the deficit Δv_s increases most rapidly (to $\sim$100 m/sec) for particles with $d_{av} = 190\,\mu$ and $\rho_r = 8.6$ g/cm^3, for which the relaxation time τ_D is a maximum (compared with the other particles). Figure 3 shows that Δv_s varies over wide limits from $\Delta v_s \simeq 130$ m/sec for lycopodium particles ($d_{av} = 25\,\mu$ $\rho_r = 0.5$ g/cm^3) to 430 m/sec for bronze particles ($d_{av} = 190\,\mu$, $\rho_r = 8.6$ g/cm^3).

The drag coefficient C_D was calculated from Eq. (1.3). An estimate of the relative error in determining C_D, allowing for the measurement errors of v, v_r, $\partial v_r/\partial z$, and d_r gives a value $\leqslant$10%. The Reynolds numbers in these tests fell in the range $4 \cdot 10^2$-$2.2 \cdot 10^4$, and the Mach number fell in the range 0.5-2.1; here the flow conditions varied from subsonic (for lycopodium particles with $d_{av} = 25\,\mu$, $\rho_r = 0.5$ g/cm^3) to supersonic (bronze and lucite particles). Since Re and M_s varied from one measurement point to another, it was expedient to organize the data for C_D in the form of a correlation $C_D = f(M_s)$, since the influence of Re on C_D in the range $Re = 0.4 \cdot 10^3$-$2.2 \cdot 10^4$, as can be seen from the "standard" resistance curve [1], is small (C_D varies smoothly in this region from 0.6 at $Re = 0.4 \cdot 10^3$ to $\sim$0.5 at $Re = 2.2 \cdot 10^4$). In addition, from the experimental data on flow over bodies in wind tunnels [8, 9], it is known that the influence of Re on C_D decreases with increase of M, and for $M \geqslant 0.8$ the curves $C_D = f(M)$ practically coincide for different Re [9].

Figure 4 shows the drag coefficients C_{exp}, obtained in this work, as a function of $M_s = (v - v_r)/a$; the solid curves are labeled as follows: I is the drag coefficient of a sphere C_T as a function of the oncoming stream Mach number (M) taken from [8] and obtained from results of measurements in studies of the flow over a fixed sphere in wind tunnels and in free flight tests; and II is the relation [2]

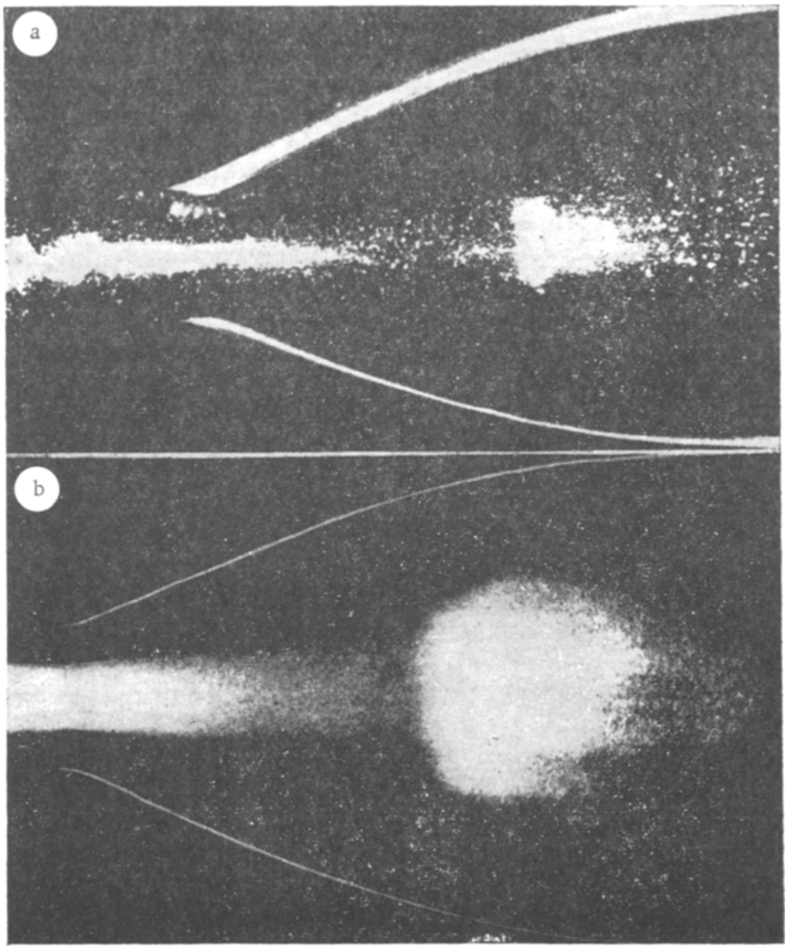

Fig. 6

$$C_D = C_D^0 \frac{1 - 0.445M + 4.84M^2 - 9.73M^3 + 6.94M^4}{(1 + 1.2MC_D^0)^{1/2}}, \tag{3.1}$$

obtained by approximation to the experimental data of [10], and referring to fine particles moving in a gas stream, accounting for the effects of compressibility, inertia, and rarefaction [11] in the range M = 0.2-1.0 (the value C_D^0 is determined for a given Re from the "standard" resistance curve). It can be seen from Fig. 4 that the values of C_{exp} lie in the range C_{exp} = 0.6-2.3, and with increase of M_s, in analogy with the dependence of I (see Fig. 4), there is an increase in the drag coefficient from C_{exp} = 0.6-1.1 at M_s = 0.5-0.7 to C_{exp} = 1.5-2.3 at M_s = 1.7-2.2. A comparison of the absolute values of C_{exp} and C_T, given by curve I, shows that $C_{exp} \geqslant C_T$, and their relative divergence ($\eta_{max} = C_{exp}/C_T \simeq 2.3$) and also the scatter of the experimentally determined values of C_{exp} for the various particles (at the same value of M_s) are apparently due to the considerable difference in the surface state and particle shape, referring to different fractions (particles of bronze, lycopodium, aluminum, and lucite).

4. We now examine the results of experiments dealing with investigation of the phenomenon of high-speed nonequilibrium of a gas and particles, appearing when a two-phase flow passes through a normal density shock. The passage of fine particles through a region with a large gas velocity gradient allows us an efficient method of investigating their inertial properties and makes shock waves a convenient object for the study of nonequilibrium processes in a mixture of gas and particles and, in particular, for choosing aerosols which will accurately follow the gas stream in different conditions of an aerodynamic experiment, which is important in the development of the laser-Doppler method of gas velocity measurement [12, 13]. The main objectives of this paper are to investigate the nature of the velocity varia-

tion of different particles in the relaxation zone, to determine their aerodynamic drag in order to have further verification of the results stated above, to estimate the size of the smoke particles with the lowest inertia in the experimental data, and to analyze their possible use to measure gas velocity by the laser-Doppler method.

Figures 5 and 6 show the experimental results illustrating certain features of the passage of a two-phase flow through a normal density shock, formed in the supersonic (expanding) part of a two-dimensional shaped nozzle in a nonequilibrium operating regime [14]. The dimensions of the throat were: height, $h = 2.6$ mm; width, $l = 20$ mm; and the Mach number ahead of the shock is 2.8. Figure 5 shows the data on velocity measurement for the various particles [1, gas (calculation); 2, smoke particles; 3, polystyrene particles, $d_r = 1\text{-}3\ \mu$, $\rho_r = 1.02$ g/cm^3; 4, lycopodium particles $d_{av} = 25\ \mu$, $\rho_r = 0.5$ g/cm^3; 5, bronze particles $d_{av} = 50\ \mu$, $\rho_r = 8.6$ g/cm^3; 6, zone of density shock oscillations]. Figure 6a, b shows typical photographs obtained by the high-speed laser visualization method [5, 6] (a, polystyrene particles $d_r = 1\text{-}3\ \mu$ and bronze particles $d_{av} \approx 80\ \mu$ are simultaneously introduced into the stream, the length of the laser radiation pulse is ≈30 nsec; b, chalk particles $d_r = 1\text{-}10\ \mu$, with the peak operating conditions of the laser).

Figure 5 shows the values of gas velocity and also of temperature T and density ρ, obtained by computation on a one-dimensional model of steady isentropic flow with given stagnation parameters ($p_0 = 8$ atm, $T_0 = 260$°K) and the nozzle profile, and also accounting for the well-known relations [15] for a direct density shock (in the region located directly behind the shock wave). We note that the mass fraction of particles in the tests described did not exceed 1%. In this case, as previous tests have shown, we can neglect the influence of the particles on the gas parameters and assume that the latter do not vary from their values in the clean flow. (An increase in the velocity of smoke particles and therefore in the velocity of the gas as one goes away from the shock is associated with compression of the flow due to a reduction in the cross section toward the nozzle exit.) It can be clearly seen that the velocity of fine particles (polystyrene smoke) falls sharply behind the shock (see Fig. 5), which leads to a rapid increase in the concentration of these particles (see Fig. 6a, b), while the velocity of bronze particles ($d_{av} \approx 50\ \mu$, $\rho_r = 8.7$ g/cm^3) and their concentration practically does not change (see Figs. 5, 6a).

A simple estimate shows [16] that when particles pass through a shock wave, the influence of the force due to pressure gradient in the shock wave front $F_p = {}^4/_3\pi r_r^3 \partial p/\partial z$ [1] can be neglected in comparison with the influence of the aerodynamic drag force F_D on the particles, since the flight time of the particle through the shock wave front $t = \delta/v_r \leqslant 10^{-8}$ sec, which is considerably less than the characteristic relaxation time $\tau_D = 4\rho_r d_r/3C_D\rho(v - v_r)$ [1] due to the influence of F_D. This indicates that particle stagnation occurs immediately behind the shock wave (in the relaxation zone) under the influence of F_D, and allows one to write down an expression for the drag coefficient in the form $C_D = [4\rho_r d_r v_r/3\rho(v - v_r)^2]\partial v_r/\partial z$ (1.3). To determine C_D in the relaxation zone the most convenient of all the aerosols used in the experiment were the lycopodium particles ($d_{av} = 25\ \mu$, $\rho_r = 0.5$ g/cm^3), which had quite a narrow size spectrum $\Delta d_r = 1\text{-}2\ \mu$ and a smooth dependence $\partial v_r/\partial z$. For these particles we obtained a value $C_D \approx 0.9$ for $M_s \approx 0.6$ and $Re \approx 10^3$, which agrees satisfactorily with results presented earlier on the measurement of C_D for these particles under conditions of acceleration in a supersonic nozzle and also with the data of [17]. The accuracy in determining C_D is $\leqslant 10\%$. The difference between the experimentally determined drag coefficient is ∿80% of the value C_D^0, corresponding to the "standard" drag curve [1] ($C_D^0 \approx 0.5$ for $Re \approx 10^3$), and may be due in part to the influence of $M_s \approx 0.6$, and also to the particle roughness and shape, which differed substantially from a perfect spherical surface [17].

The investigation of the inertial properties of smoke particles has shown (see Fig. 5), that the velocity of the latter, to within the measurement accuracy of 1-2%, agrees with the theoretically calculated gas velocity, both before and after the shock wave. This allows us to estimate the aerosol size from comparison with the theoretical curves $v_r = f(z)$ obtained in [12] for a density shock ($M = 3$, $p_0 = 1$ atm, $T_0 = 294$°K). Taking into account the effect of shock oscillation, due to fluctuations in the velocity of the oncoming stream and the limiting accuracy in measurement of the distance from the point probed to the shock ($\Delta z \approx 1$ mm), we can conclude that the aerosols tested have a size $d_r < 0.2\text{-}0.5\ \mu$, and because their inertia is low, we can apply them successfully to investigate gas streams using the laser-Doppler method.

It can be seen from Fig. 6 that when fine particles (particles of chalk or polystyrene, d_r = 1-10μ) pass through a density shock, they acquire a transverse velocity leading to a rapid transverse expansion of the region occupied by the particles (see Fig. 6b). This experimental fact, which is difficult to explain only by collisions of particles because their concentration is quite small $\leqslant$ 1% by weight), requires further study.

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